Augmentation to the Extended Kalman-Bucy Filter for Single Target Tracking

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Abstract – *Tracking agile aircraft under high accelerations generally demands sophisticated models for determining trajectories with desirable dynamics and accuracy. Often this raises complexity of the estimation algorithm as it gives rise to more elaborated methods for both taking model nonlinearities into account and handling a greater number of state variables that describe the model. The approach of this work recalls a 3D model based on flight dynamics of a point of mass for which augmentation to the Extended Kalman-Bucy filter (EKBF) is proposed. Two methods of augmentation to the EKBF filter are studied: (i) use of second-order terms to approximate the model according to Daum's theory; (ii) deployment of a neural network coupled to the filter for compensation of modeling and calculation errors. The evaluation of the filters performance is accomplished by measuring nonlinearities, bias, accuracy and robustness. The designed filters are suitably accurate and robust for tracking targets in air combat scenario.*

Keywords: Target tracking, filtering, estimation, nonlinear filter, Kalman Filter, aircraft.

1 Introduction

1.1 Overview

For more than two decades the Extended Kalman Filter (EKF) has been the most applied approximate nonlinear filter for practical nonlinear estimation problems. Solutions based on the EKF rely on its implementation simplicity at the cost of dealing with its main disadvantage, which comes from the necessity of the system model approximation by terms of the Taylor series for propagation of the state covariance matrix. This advantage is strictly related to the fact that analytical approximation often results in the estimator instability since part of the system dynamic characteristics are neglected on the Taylor series truncation. For the specific problem of state estimation in a continuous-time stochastic system with discrete-time measurements, the nonlinear filter equivalent to the EKF is the Extended Kalman-Bucy Filter (EKBF) which constitutes the basis for this paper.

Despite that modern realizable estimation schemes and algorithms present performance and versatility advantages over the Extended Kalman Filter, the EKF is still a good option for implementation simplicity and low computational capacity demand.

This paper addresses the nonlinear filtering problem using a simple but efficient continuous-time aircraft dynamic model for which sophistication methods of the well known EKBF are developed aiming improvement of estimation accuracy and robustness. The work develops augmentation to the Extended Kalman-Bucy Filter by means of two techniques which improve the model approximation: (1) inclusion of second-order terms on model approximation by Daum's theory [1, 2, 3], and (2) compensation of modeling and calculation errors by a model-coupled neural network [4]. Thus three filters are investigated by this text: basic EKBF, EKBF augmented by technique (1) and EKBF augmented by technique (2). The context of application of such filters is the tracking phase of a fire-control radar (FCR).

The designed filters have their performance analysed according to methods of quantitative evaluation of nonlinearities, accuracy and robustness. Also evaluation of the filters performance against basic air combat maneuvers are accomplished.

1.2 Text organization

This paper is organized as follows. Section 1 was dedicated to introductory overview. Section 2 presents the adopted model used as basis for development of the studied filters. Section 3 presents the theoretical basis and considerations for the development of the proposed nonlinear filters. Section 4 presents the methods for implementation and evaluation of the filters. Section 5 presents the results of evaluations. Section 6 establishes the main conclusions for the work.

2 Target model

2.1 Motion equations

The following presented model was developed by [5]. The derivation of this model is shown in [6].

The motion equations according to Miele's aircraft model are presented as the set of equations below. The equations were expressed with $T = \eta \cdot T_{max}$, for which T is the thrust force, T_{max} is the maximum available thrust at any phase of flight and η is the percentage of utilized thrust:

$$\begin{cases} \dot{x} &= v \cdot \cos\gamma \cdot \cos\chi \\ \dot{y} &= v \cdot \cos\gamma \cdot \sin\chi \\ \dot{h} &= v \cdot \sin\gamma \\ \dot{\gamma} &= \frac{1}{m \cdot v} \left\{ [L + \eta \cdot T_{max} \cdot \sin\alpha] \cos\mu - m \cdot g \cdot \cos\gamma \right\} \\ \dot{\chi} &= \frac{1}{m \cdot v \cdot \cos\gamma} \left\{ L + \eta \cdot T_{max} \cdot \sin\alpha \right\} \sin\mu \\ \dot{v} &= \frac{1}{m} \left\{ \eta \cdot T_{max} \cdot \cos\alpha - D - m \cdot g \cdot \sin\gamma \right\} \end{cases} \tag{1}$$

For this set of equations, the state variables are the horizontal coordinates x and y, the altitude h, the flight path angle γ, the heading angle χ and the velocity v. The control variables of this model are the angle of attack α, the angle of roll (bank) μ and the percentage of thrust η.

2.2 Aerodynamic model

The aerodynamic forces are described as follows:

$$\begin{aligned} L &= \bar{q}(h, v) \cdot S_{ref} \cdot C_L(\alpha, M(h, v)) \\ D &= \bar{q}(h, v) \cdot S_{ref} \cdot C_D(\alpha, M(h, v)) \end{aligned} \tag{2}$$

L, D are the lift and drag forces respectively; C_L, C_D are the total coefficients of lift and drag; $\bar{q}(h, v) = 0.5\rho(h) \cdot v^2$ is the dynamic pressure; ρ is the air density; S_{ref} is a reference area (wing planform surface); $\bar{c}$ is the mean aerodynamic chord. The standard atmosphere was considered for calculation of ρ in all equations.

The total aerodynamic coefficients are generally proposed by [7], but for the purposes of this work a simplified form was adopted:

$$C_L \approx C_{L_0} + \left(C_{L_\alpha} + C_{L_{\delta e}} K_{\frac{\delta e}{\alpha}} \right) \alpha \tag{3}$$

$$C_D \approx C_{D_0} + \left(C_{D_\alpha} + C_{D_{\delta e}} K_{\frac{\delta e}{\alpha}} \right) \alpha \tag{4}$$

As the aircraft motion equations describe the dynamics of a point of mass, there is no prescribed state of surface control and, therefore, the portion that depends on the elevator position ($C_{L_{\delta e}}\delta_e$ and $C_{D_{\delta e}}\delta_e$) cannot be explicitly calculated. Nevertheless it is known that a fraction of the angle of attack at a certain instant is consequence of the elevator deflection, for which there is a correspondence between the elevator deflection and the current steady-state of the angle of attack $C_{L_{\delta e}}\delta_e = C_{L_{\delta e}} K_{\frac{\delta e}{\alpha}} \alpha$.

2.3 Thrust model

The standard thrust model used for the development of the filters was the one that describes a turbojet engine or a turbofan with low bypass ratio:

$$T_{max} = T_{sl} \cdot \left(\frac{\rho}{\rho_{sl}} \right)^{0.7} \tag{5}$$

3 Nonlinear filters

3.1 Extended Kalman-Bucy Filter

The target trajectory is described by the variables x, y and h, which shall be estimated. The states vector is then defined as $\mathbf{x} = [x, y, h, \gamma, \chi, v, \alpha, \mu, \eta]^T$. Let the state and output equations of a hybrid Gaussian stochastic system be described by the stochastic differential equation of Itô [8] and by the measurement equation respectively:

$$d\mathbf{x}(t) = \mathbf{f}(\mathbf{x}(t), t)\, dt + G(t) d\mathbf{w}(t), \quad \forall t \in \mathcal{R} \tag{6}$$

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k, t_k) + \mathbf{v}_k, \tag{7}$$

where $\mathbf{x}(t)$ is the states vector and $\mathbf{z}_k$ is the measurements vector. The state error noise is a Wiener-Levy process for which $d\mathbf{w}(t)$ is an increment. The vector $\mathbf{v}_k$ is the measurement noise. The process noise and the measurement noise are mutually independent and independent on the initial state $\mathbf{x}_0$ whose probability density $p(\mathbf{x}_0)$ is determined.

When the system is linear with assumed Gaussian probability densities for the states and noises, the functions $\mathbf{f}(\mathbf{x}(t), t)$ and $\mathbf{h}(\mathbf{x}_k, t_k)$ are linear on the states, and the solution to the estimation problem is the Kalman-Bucy Filter. If the system is nonlinear, the probability densities for filtering and prediction may be approximated by Gaussian and reproducible densities such that their moments can be recursively evaluated by a set of equations approximately linear in relation to the the mean $\hat{\mathbf{x}}$ and the covariance matrix $\mathbf{P}$. Those approximately linear equations define the Extended Kalman-Bucy Filter.

For the filtering step, the statistical moments of the approximately Gaussian $p(\mathbf{x}_k|\mathbf{Z}_k)$, i.e. the mean $\hat{\mathbf{x}}_k$ and the covariance matrix $\mathbf{P}_k$ at a given instant of time t_k are calculated by

$$\begin{gathered} \hat{\mathbf{x}}_k = \hat{\mathbf{x}}'_k \\ + \mathbf{P}'_k \mathbf{H}^T(\hat{\mathbf{x}}'_k, t_k)\, \mathbf{K}(\hat{\mathbf{x}}'_k, t_k) \left[\mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}'_k, t_k) \right] \end{gathered} \tag{8}$$

$$\begin{gathered} \mathbf{P}_k = \mathbf{P}'_k \\ - \mathbf{P}'_k \mathbf{H}^T(\hat{\mathbf{x}}'_k, t_k)\, \mathbf{K}(\hat{\mathbf{x}}'_k, t_k)\, \mathbf{H}(\hat{\mathbf{x}}'_k, t_k)\, \mathbf{P}'_k \end{gathered} \tag{9}$$

$$\mathbf{K}(\hat{\mathbf{x}}'_k, t_k) = \left[\mathbf{H}(\hat{\mathbf{x}}'_k, t_k)\, \mathbf{P}'_k \mathbf{H}^T(\hat{\mathbf{x}}'_k, t_k) + \mathbf{R}_k \right]^{-1} \tag{10}$$

For prediction step, the statistical moments of the approximately Gaussian $p\left(\mathbf{x}(t)|\mathbf{Z}_k\right)$, i.e. the mean $\hat{\mathbf{x}}'(t)$ and the covariance matrix $\mathbf{P}'(t)$ for a continuous interval $t \in I_{k,\,k+1}$ satisfy the following ordinary differential equations:

$$\frac{d\hat{\mathbf{x}}'(t)}{dt} = \mathbf{f}\left(\hat{\mathbf{x}}'(t), t\right) \tag{11}$$

$$\frac{d\mathbf{P}'(t)}{dt} = \mathbf{F}\left(\hat{\mathbf{x}}'(t), t\right)\mathbf{P}'(t) + \mathbf{P}'(t)\mathbf{F}^T\left(\hat{\mathbf{x}}'(t), t\right) + \mathbf{Q}(t) \tag{12}$$

It is assumed that the time rate of change of angle of attack $\{\dot{\alpha}\}$, roll angle $\{\dot{\mu}\}$, and thrust rating $\{\dot{\eta}\}$ constitute independent Gaussian processes with zero mean and determined variances $\sigma^2_{\dot{\alpha}}$, $\sigma^2_{\dot{\mu}}$ and $\sigma^2_{\dot{\eta}}$ thus $\mathbf{Q}(t)$ is a diagonal matrix of the following form $\mathbf{Q} = diag\left(\left[0,\, 0,\, 0,\, 0,\, 0,\, 0,\, \sigma^2_{\dot{\alpha}},\, \sigma^2_{\dot{\mu}},\, \sigma^2_{\dot{\eta}}\right]\right)$. The measurements are values of x, y and h obtained at each $t = t_k$ such that the measurement equation is of the linear type $\mathbf{z}_k = \mathbf{H}\cdot\mathbf{x}_k + \mathbf{v}_k$ for which $\mathbf{h}\left(\mathbf{x}_k, t_k\right) = \mathbf{H}\cdot\mathbf{x}_k$ where $\mathbf{H}\left(\hat{\mathbf{x}}'_k, t_k\right) = \left[\begin{array}{cc} \mathbb{I}_{3\times3} & \mathbf{0}_{3\times6}\end{array}\right]$. $\mathbf{R}_k$ is the covariance matrix of the sensor noise. $\mathbf{F}\left(\hat{\mathbf{x}}'(t), t\right)$ is the jacobian matrix of $\mathbf{f}\left(\mathbf{x}(t), t\right)$ evaluated at $\hat{\mathbf{x}}'(t)$.

3.2 EKBF - Daum's theory

In [1], Daum assumes the existence of a gradient function $\mathbf{r} = \frac{\partial}{\partial\mathbf{x}}\left[\ln\mathbf{\Psi}\left(\mathbf{x}, t\right)\right]$ and proposes some conditions such that if any proposed function $\mathbf{\Psi}\left(\mathbf{x}, t\right)$ satisfies them, then an exact linear filter may be described. The conditions are stated as follows.

Condition 1: $\mathbf{\Psi}\left(\mathbf{x}, t\right)$ satisfies the Fokker-Planck equation for the interval $(t_{k-1},\, t_k)$:

$$\frac{\partial\mathbf{\Psi}(\mathbf{x},t)}{\partial t} = -\left(\frac{\partial\mathbf{\Psi}}{\partial\mathbf{x}}\right)\mathbf{f} - \mathbf{\Psi}\left[\text{trace}\left(\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right)\right] + \frac{1}{2}\text{trace}\left(\mathbf{Q}\frac{\partial^2\mathbf{\Psi}}{\partial\mathbf{x}^2}\right) \tag{13}$$

Condition 2:

$$\text{trace}\left(\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right) + \frac{1}{4}\mathbf{r}\mathbf{Q}\mathbf{r}^T = \mathbf{x}^T\mathbf{A}\mathbf{x} + \mathbf{b}^T\mathbf{x} + c \tag{14}$$

for a symmetric matrix $\mathbf{A}$, vector $\mathbf{b}$, and a scalar c.

Condition 3:

$$\mathbf{f} - \frac{1}{2}\mathbf{Q}\mathbf{r}^T = \mathbf{D}\mathbf{x} + \mathbf{E} \tag{15}$$

for some matrix $\mathbf{D}$ and some vector $\mathbf{E}$.

It is possible to prove that if $\mathbf{\Psi}$ exists and satisfies all the previous conditions then a non normalized density function at an instant t_k for a set of measurements $\mathbf{Z}_k$ can be described by

$$p\left(\mathbf{x}_k, t_k\right) = \mathbf{\Psi}\left(\mathbf{x}_k, t_k\right)^{\frac{1}{2}}\exp\left\{-\frac{1}{2}\left[\mathbf{x}_k - \mathbf{m}_k\right]^T\mathbf{M}_k^{-1}\left[\mathbf{x}_k - \mathbf{m}_k\right]\right\} \tag{16}$$

As this density function solves the Fokker-Planck equation, the parameters $\mathbf{m}$ and $\mathbf{M}$ are propagated between any two subsequent observations according to a set of differential ordinary equations very similar to the prediction equations of the EKBF. The parameters $\mathbf{m}$ and $\mathbf{M}$ are not the mean and the covariance matrix of the states, but are similar to them. The approach for solving the practical difficulty of using $\mathbf{m}$ and $\mathbf{M}$ to define a filter was proposed by Schmidt [3], who comes up with equivalent conditions a function $\mathbf{f}$ must satisfy for approximately complying with conditions 1, 2 and 3.

For solving the Daum's proposition, the function $\mathbf{f}$ which does not depend explicitly on time may be written as:

$$\mathbf{f}\left(\mathbf{x}\right) = \hat{\mathbf{n}} + \mathbf{B}\mathbf{x} + \mathbf{U} + \mathbf{O}\left(3\right) \tag{17}$$

where $\hat{\mathbf{n}} = \mathbf{f}\left(\mathbf{x}'\right)$, $\mathbf{B} = \frac{\partial\mathbf{f}}{\partial\mathbf{x}}\Big|_{\mathbf{x}=\hat{\mathbf{x}}'}$ $\mathbf{U} = \left[\mathbf{x}^T\mathbf{G}_i\mathbf{x}\right]_{9\times1}$. The portion $\mathbf{O}\left(3\right)$ denotes all terms of third and higher orders. The matrices $\mathbf{G}_i$ are symmetric and calculated by $\mathbf{G}_i = \frac{\partial}{\partial\mathbf{x}}\left(\frac{\partial f_i(\mathbf{x})}{\partial\mathbf{x}}\right)^T = \frac{\partial}{\partial\mathbf{x}}\left(\frac{\partial\dot{x}_i}{\partial\mathbf{x}}\right)^T\Big|_{\mathbf{x}=\hat{\mathbf{x}}'}$. In addition, the expression

$$\text{trace}\left(\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right) = d + \mathbf{S}^T\mathbf{x} + \mathbf{x}^T\mathbf{L}\mathbf{x} + \mathbf{O}\left(3\right) \tag{18}$$

shall be valid for some symmetric matrix $\mathbf{L}$, a vector $\mathbf{S}$ and a scalar d. If conditions 2 and 3 are true including terms of order up to second, then $\mathbf{\Psi}\left(\mathbf{x}, t\right) = \exp\left\{h\left(\mathbf{x}, t\right)\right\}$ such that $h\left(\mathbf{x}, t\right) = \mathbf{x}^T\mathbf{V}\mathbf{x} + \mathbf{g}^T\mathbf{x} + \alpha\left(t\right)$ for a symmetric matrix $\mathbf{V}$, a vector $\mathbf{g}$ and a scalar function $\alpha\left(t\right)$. Thus, in order to comply with condition 1 approximately, the following equivalent conditions may be set:

Conditions A:

$$2\left(\mathbf{V}\mathbf{Q}\mathbf{V} - \mathbf{V}\mathbf{B}\right) = \mathbf{L} + g_1\mathbf{G}_1 + \cdots + g_n\mathbf{G}_n \tag{19}$$

$$\mathbf{g}^T\left(2\mathbf{Q}\mathbf{V} - \mathbf{B}\right) = \mathbf{S}^T + 2\hat{\mathbf{n}}^T\mathbf{V} \tag{20}$$

$$\dot{\alpha}\left(t\right) = \frac{1}{2}\text{trace}\left(\mathbf{Q}\mathbf{g}\mathbf{g}^T + 2\mathbf{Q}\mathbf{V}\right) - \mathbf{g}^T\hat{\mathbf{n}} - d \tag{21}$$

If $\mathbf{f}$ is linear or approximately linear, then the expressions (19) and (20) are trivially satisfied with $\mathbf{V} = 0$ and $\mathbf{g} = 0$. The Kalman Filter and the Extended Kalman Filter are contained in this category of solutions. Another category of solutions can be obtained for a weaker set of conditions, which is developed based on considerations over higher order terms of an approximate solution to the equation (13):

Conditions B:

$$2\mathbf{VQV}-\left(\mathbf{VB}+\mathbf{B}^T\mathbf{V}\right)= \\ \left[\mathbf{L}+\left(g_1\mathbf{G}_1+\cdots+g_n\mathbf{G}_n\right)\right] \quad (22)$$

$$\mathbf{g}^T\left(2\mathbf{QV}-\mathbf{B}\right)=\mathbf{S}^T+2\hat{\mathbf{n}}^T\mathbf{V} \quad (23)$$

$$\ddot{\alpha}\left(t\right)=\beta\left(t\right) \quad (24)$$

The relation between the parameters $\mathbf{M}$ and $\mathbf{m}$, and $\mathbf{P}$ and $\hat{\mathbf{x}}$, are established based on an equivalence between the density (16) and a density of the form $C_2\exp\left\{-\frac{1}{2}\left[\mathbf{x}-\hat{\mathbf{x}}\right]^T\mathbf{P}^{-1}\left[\mathbf{x}-\hat{\mathbf{x}}\right]\right\}$. Thus, the correspondences $\mathbf{P}^{-1}=\mathbf{M}^{-1}-\mathbf{V}$ and $\hat{\mathbf{x}}=\mathbf{PM}^{-1}\mathbf{m}+\frac{1}{2}\mathbf{Pg}$ are valid.

By transforming the equations of prediction for $\mathbf{M}$ and $\mathbf{m}$ according to [3], the following expressions are achieved:

$$\dot{\mathbf{P}}=\mathbf{BP}+\mathbf{PB}^T+\mathbf{Q} \\ -\mathbf{P}\left(2\cdot\mathbf{L}+g_1\mathbf{G}_1+\cdots+g_n\mathbf{G}_n\right)\mathbf{P} \quad (25)$$

$$\dot{\hat{x}}=\hat{\mathbf{n}}+\mathbf{B}\hat{\mathbf{x}} \\ -\mathbf{P}\left(2\cdot\mathbf{L}+g_1\mathbf{G}_1+\cdots+g_n\mathbf{G}_n\right)\hat{\mathbf{x}}-\mathbf{PS} \quad (26)$$

By transforming the updating equations for $\mathbf{M}$ and $\mathbf{m}$, the equivalent updating equations are:

$$\mathbf{P}_{k(+)}^{-1}\hat{\mathbf{x}}_{k(+)} = \mathbf{H}_k^T\mathbf{R}_k^{-1}\mathbf{z}_k+\mathbf{P}_{k(-)}^{-1}\hat{\mathbf{x}}_{k(-)} \quad (27)$$

$$\mathbf{P}_{k(+)}^{-1} = \mathbf{H}_k^T\mathbf{R}_k^{-1}\mathbf{H}_k-\mathbf{P}_{k(-)}^{-1} \quad (28)$$

In order to comply with all the conditions established by Daum [1] and refined by Schmidt [3] it is convenient to establish generic forms $\mathbf{V}=\left[v_{j\,k}\right]_{9\times9}$, $\mathbf{g}=\left[g_j\right]_{9\times1}$, $\mathbf{n}=\left[n_j\right]_{9\times1}$, $\mathbf{B}=\left[b_{j\,k}\right]_{9\times9}$, and $\mathbf{G}_i=\left[\gamma_{i,\,j\,k}\right]_{9\times9}$, where j is the index of rows and k is the index of columns, and i is an index relative to any of the state variables $\mathbf{x}=\left[x_j\right]_{9\times1}$.

Due to the complexity of function $\mathbf{f}$, a simple solution to the filter of Daum requires compliance with the **conditions B** and the simplifications $\frac{\partial\rho}{\partial h}\approx0$ and $\frac{\partial}{\partial h}\left(\frac{\partial\rho}{\partial h}\right)\approx0$ for obtention of vector $\mathbf{S}=\left[s_j\right]_{9\times1}$ and of the symmetric matrix $\mathbf{L}=\left[l_{j\,k}\right]_{9\times9}$. The result of approximately satisfying (18) is:

$$d=\left[\frac{g}{v}\sin\gamma-\frac{\rho}{m}\cdot v\cdot S_{ref}C_{D_\alpha eq}\alpha\right]_{\mathbf{x}=\hat{\mathbf{x}}'} \quad (29)$$

$$\mathbf{S}=\begin{bmatrix}0\\0\\0\\\frac{g}{v}\cos\gamma\\0\\-\left(\frac{g}{v^2}\sin\gamma+\frac{\rho}{m}\cdot S_{ref}C_{D_\alpha eq}\alpha\right)\\-\frac{\rho}{m}\cdot v\cdot S_{ref}C_{D_\alpha eq}\\0\\0\end{bmatrix}_{\mathbf{x}=\hat{\mathbf{x}}'} \quad (30)$$

The non null elements of $\mathbf{L}$ are $l_{44}=-\frac{g}{v}\sin\gamma$, $l_{46}=l_{64}=-\frac{g}{v^2}\cos\gamma$, $l_{66}=+\frac{2g}{v^3}\sin\gamma$ and $l_{67}=l_{76}=-\frac{\rho}{m}\cdot S_{ref}C_{D_\alpha eq}$. The first relation of conditions B results in a vector $\mathbf{g}=\left[0,\,0,\,g_3,\,0,\,0,\,0,\,0,\,0,\,0\right]^T$ and a matrix $\mathbf{V}=\left[v_{j\,k}\right]_{9\times9}$ with non null elements determined by:

$$v_{44} = -\frac{l_{44}+g_3\cdot\gamma_{3,\,44}}{2b_{44}} \quad (31)$$

$$v_{46} = -\frac{l_{46}+g_3\cdot\gamma_{3,\,46}}{\left(b_{46}-b_{64}\right)} \quad (32)$$

$$v_{66} = -\frac{l_{66}}{2b_{66}} \quad (33)$$

$$v_{67} = -\frac{l_{67}}{2b_{67}} \quad (34)$$

The second relation of conditions B leads to:

$$v_{37} = -\frac{n_6}{n_3}v_{67} \quad (35)$$

$$v_{33} = -\frac{n_7}{n_3}v_{37} \quad (36)$$

$$g_3 = -\frac{\left[\frac{n_4b_{36}}{b_{44}}l_{44}+2\frac{\left(n_6b_{36}-n_4b_{34}\right)}{\left(b_{46}+b_{64}\right)}l_{46}\right]}{\left[\frac{n_4b_{36}}{b_{44}}\gamma_{3,\,44}+2\frac{\left(n_6b_{36}-n_4b_{34}\right)}{\left(b_{46}+b_{64}\right)}\gamma_{3,\,46}\right]} \\ -\frac{b_{34}\left(s_6+2n_6v_{66}+2n_7v_{67}\right)-b_{36}s_4}{\left[\frac{n_4b_{36}}{b_{44}}\gamma_{3,\,44}+2\frac{\left(n_6b_{36}-n_4b_{34}\right)}{\left(b_{46}+b_{64}\right)}\gamma_{3,\,46}\right]} \quad (37)$$

where $\gamma_{3,\,44}=-v\sin\gamma$ and $\gamma_{3,\,46}=\cos\gamma$, and the relations (31) and (32) are calculated based on (37). As $\dot{\alpha}'=\left[-g_3\cdot n_3-d\right]_{\mathbf{x}=\hat{\mathbf{x}}'}$ according to the definition by (21) one can conclude that $\ddot{\alpha}'=0$ for satisfying the last relation of conditions B. The second-order terms which increment the prediction equations of the EKBF can be easily computed based on $\mathbf{L}$, $\mathbf{g}$ and $\mathbf{S}$.

3.3 EKBF - neural network

In this section, the Extended Kalman-Bucy Filter modified to comprise a neural network, herein designated by Neural Extended Kalman-Bucy Filter (NEKBF), is developed based on [4]. The design of the NEKBF consists in determining the filtering and prediction equations for an augmented state vector which contains the coefficients of a neural network. The neural network has the functionality of calculation of deviations on the time derivative of the state variables, which are compensated to reduce the effects of modeling, approximation and computation errors. The neural network training is accomplished by the recursive estimation of the network coefficients. The figure 1 depicts a block diagram of a NEKBF, for compensation of the deviations on the state variables.

For the particular case of this work, the neural network is conceived to compensate the deviations on the velocities $\dot{x}$, $\dot{y}$ and $\dot{h}$ based on variables which affect

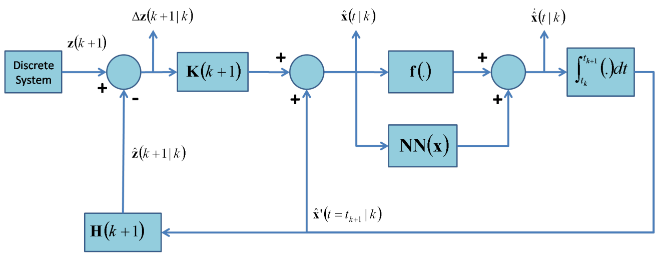

Figure 1: Block diagram representing the NEKBF

directly the target trajectory dynamics: γ, χ and v. Defining the neural network in vectorial notation:

$$\mathbf{NN} = \mathbf{W}_O \cdot \mathbf{g}\left(\mathbf{W}_I \cdot \mathbf{x} + \mathbf{b}_I\right) + \mathbf{b}_O \tag{38}$$

The matrices of input and output coefficients $\mathbf{W}_I$ and $\mathbf{W}_O$ respectively, and the input and output *bias* vectors $\mathbf{b}_I$ and $\mathbf{b}_O$ are defined as:

$$\mathbf{W}_I = \begin{bmatrix} \mathbf{0}_{3\times 3} & \begin{matrix} w_{i,11} & w_{i,12} & w_{i,13} \\ w_{i,21} & w_{i,22} & w_{i,23} \\ w_{i,31} & w_{i,32} & w_{i,33} \end{matrix} & \mathbf{0}_{3\times 3} \end{bmatrix} \tag{39}$$

$$\mathbf{W}_O = \begin{bmatrix} w_{o,11} & w_{o,12} & w_{o,13} \\ w_{o,21} & w_{o,22} & w_{o,23} \\ w_{o,31} & w_{o,32} & w_{o,33} \\ & \mathbf{0}_{6\times 3} & \end{bmatrix} \tag{40}$$

$$\mathbf{b}_I = \left[\begin{array}{ccc} b_{i,1} & b_{i,2} & b_{i,3} \end{array}\right]^T \tag{41}$$

$$\mathbf{b}_O = \left[\begin{array}{ccc} b_{o,1} & b_{o,2} & b_{o,3} \end{array}\right]^T \tag{42}$$

The vectorial function $\mathbf{g}(.)$ describes the dynamic function of the hidden layer neurons defined as the logarithmic sigmoid $\mathbf{g}(\mathbf{u}) = \left[\begin{array}{ccc} \frac{1}{1+\exp\left(-\frac{u_1}{\tau_1}\right)}, & \frac{1}{1+\exp\left(-\frac{u_2}{\tau_2}\right)}, & \frac{1}{1+\exp\left(-\frac{u_3}{\tau_3}\right)} \end{array}\right]^T$ where $\mathbf{u} = \mathbf{W}_I \cdot \mathbf{x} + \mathbf{b}_I$ and $\tau_i = 1$.

The Extended Kalman-Bucy Filter shall couple the network training to the states estimation, i.e. achieving the weight factors and the bias portions of the neural network together with the estimation of the state variables simultaneously. In order to do that the augmented state vector must comprise all parameters to be recursively estimated:

$$\mathbf{x}_A(t) = \begin{bmatrix} \mathbf{x}(t) \\ \cdots\cdots \\ \bar{\omega}_i(t) \\ \bar{\beta}_i(t) \\ \bar{\omega}_o(t) \\ \bar{\beta}_o(t) \end{bmatrix} \tag{43}$$

for $\bar{\omega}_i(t)$ is a vector with the network input weights (non null elements of $\mathbf{W}_I$) , $\bar{\beta}_i(t) = \mathbf{b}_I$, $\bar{\omega}_o(t)$ is a vector with the network output weights (non null elements of $\mathbf{W}_O$), and $\bar{\beta}_o(t) = \mathbf{b}_O$.

The augmented target dynamic function must assume the form:

$$\mathbf{f}_A\left(\mathbf{x}_A(t), t\right) = \begin{bmatrix} \mathbf{f}\left(\mathbf{x}(t), t\right) + \mathbf{NN}\left(\mathbf{x}, \bar{\omega}_i, \bar{\beta}_i, \bar{\omega}_o, \bar{\beta}_o\right) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \mathbf{0}_{24\times 1} \end{bmatrix} \tag{44}$$

The jacobian of the expanded dynamic function is:

$$\mathbf{F}_A\left(\hat{\mathbf{x}}'_A(t), t\right) \triangleq \left.\frac{\partial \mathbf{f}_A(\mathbf{x}_A(t),t)}{\partial \mathbf{x}_A(t)}\right|_{\mathbf{x}_A(t)=\hat{\mathbf{x}}'_A(t)} = \begin{bmatrix} \mathbf{F} + \frac{\partial \mathbf{NN}}{\partial \mathbf{x}} & \vdots & \frac{\partial \mathbf{NN}}{\partial \bar{\omega}_i} & \frac{\partial \mathbf{NN}}{\partial \bar{\beta}_i} & \frac{\partial \mathbf{NN}}{\partial \bar{\omega}_o} & \frac{\partial \mathbf{NN}}{\partial \bar{\beta}_o} \\ \cdots\cdots & & \cdots\cdots & & & \\ \mathbf{0}_{24\times 9} & \vdots & \mathbf{0}_{24\times 9} & \mathbf{0}_{24\times 3} & \mathbf{0}_{24\times 9} & \mathbf{0}_{24\times 3} \end{bmatrix}_{\hat{\mathbf{x}}'_A} \tag{45}$$

As the measurement equation is linear the output matrix is of the form $\mathbf{H}_{A,k}\left(\hat{\mathbf{x}}'_{A,k}, t_k\right) = \left[\mathbf{H}_k\left(\hat{\mathbf{x}}'_k, t_k\right), \mathbf{0}_{3\times 24}\right]$.

The update and prediction equations for the NEKBF are of the same form as (8), (9), (10), (11) and (12), with the exception that the vectors and matrices have to be substituted by the augmented ones: $\hat{\mathbf{x}}_{A,k}$, $\mathbf{P}_{A,k}$, $\mathbf{H}_{A,k}$, $\mathbf{K}_{A,k}$, $\mathbf{f}_A\left(\hat{\mathbf{x}}'(t), t\right)$, $\mathbf{F}_A$, and $\mathbf{Q}_A(t)$.

The matrix $\mathbf{Q}_A(t)$ is a extended form of the process noise covariance matrix to contain the process error statistics associated to the neural network parameters. It is assumed that the time rates of change of the neural network coefficients are Gaussian stochastic processes, mutually independent and independent on the noise terms related to the plant state variables, with zero mean and given variances $\sigma^2_{\dot{\bar{\omega}}_i}$, $\sigma^2_{\dot{\bar{\beta}}_i}$, $\sigma^2_{\dot{\bar{\omega}}_o}$, $\sigma^2_{\dot{\bar{\beta}}_o}$. Thus, the matrix $\mathbf{Q}_A(t)$ is written as $\mathbf{Q}_A(t) = blkdiag\left(\mathbf{Q}(t), \mathbf{Q}_{\dot{\bar{\omega}}_i}, \mathbf{Q}_{\dot{\bar{\beta}}_i}, \mathbf{Q}_{\dot{\bar{\omega}}_o}, \mathbf{Q}_{\dot{\bar{\beta}}_o}\right)$ where $\mathbf{Q}_{\dot{\bar{\omega}}_i} = \sigma^2_{\dot{\bar{\omega}}_i}\mathbb{I}_{9\times 9}$, $\mathbf{Q}_{\dot{\bar{\beta}}_i} = \sigma^2_{\dot{\bar{\beta}}_i}\mathbb{I}_{3\times 3}$, $\mathbf{Q}_{\dot{\bar{\omega}}_o} = \sigma^2_{\dot{\bar{\omega}}_o}\mathbb{I}_{9\times 9}$ and $\mathbf{Q}_{\dot{\bar{\beta}}_o} = \sigma^2_{\dot{\bar{\beta}}_o}\mathbb{I}_{3\times 3}$. The function $blkdiag(.)$ is function for block diagonal concatenation of matrices.

4 Materials and methods

4.1 Work and validation model

The model presented on section 2 was used both as work and validation model. For the work model the parameters were assigned with typical values [6] and for the validation model the parameters were assigned with values obtained from [7, 9, 10] for each tested aircraft.

4.2 Implementation details

The prediction equations were numerically integrated between two subsequent instants of observation t_k and t_{k+1} in order to predict the state mean $\hat{\mathbf{x}}'(t)$ and the state covariance matrix $\mathbf{P}'(t)$ up to the instant $t = t_{k+1}$.

The method of numerical integration used was the fourth-order *Runge-Kutta* method with variable step.

As the sensor is a radar which measures the slant range (r), the elevation (ϕ) and the azimuth (θ) of the target, in order to provide observations and statistics of the measurement noise in cartesian coordinates the unbiased polar to cartesian transformation was applied to the radar measurements before they could be used by the filters [11]. The sensor model (radar) was assumed with the following uncertainties: $\sigma_r = 10\ m$ for slant range, $\sigma_\phi = 0.1\ deg = 0.0017\ rad$ for elevation and $\sigma_\theta = 0.1\ deg = 0.0017\ rad$ for azimuth. The sampling period of the radar was adopted as $T_{radar} = 2\ s$.

The initialization of the designed filters used a method of taking the first two measurements for computation of the state mean and covariance matrix for the instant $t = 3$ [6].

4.3 Performance indexes

For verifying accuracy, the root mean square (RMS) error is evaluated for any considered instant of time based on an adequate number N of Monte Carlo runs. For quantifying the nonlinearity the index of optimality τ_k is defined according to [12]:

$$\tau_k = \frac{1}{\sqrt{n}} \left\{ \left[\mathbf{x}_k - \hat{\mathbf{x}}_k\right]^T \mathbf{P}_k^{-1} \left[\mathbf{x}_k - \hat{\mathbf{x}}_k\right] \right\}^{\frac{1}{2}} \tag{46}$$

If τ_k is much greater than unity then the estimator nonlinearity affects the performance significantly, however if τ_k is in the proximity of one, then the estimator nonlinearity has a insignificant effect on the performance.

For checking robustness a new index was introduced [6]. It consists of a mean of the RMS errors deviations due to standardized variations on adjustable filter parameters:

$$\rho_k = \frac{1}{n \cdot p} \sum_j^n \sum_i^p \left| \epsilon_{j,\,k}|_{\pi_i + \Delta\pi_i} - \epsilon_{j,\,k} \right| \tag{47}$$

where $\epsilon_{j,\,k}$ is the RMS error of the estimate of the jth state variable at the instant t_k, π_i is the ith parameter of the filter, p is the total number of filter parameters, and n is the number of state variables. The standard variation $\Delta\pi_i$ on each filter parameter was adopted as 50% of its value tuned by design. If ρ_k is a value near zero, then the filter may be considered robust else if ρ_k is much greater than $\frac{1}{n}\sqrt{\text{trace}\left\{\mathbf{P}_k\right\}}$, then the filter is not robust. The greater the value of ρ_k the less robust is the filter.

5 Results

At first, we verify the effectiveness of the proposed EKBF against air combat maneuvers. Considering a Lockheed F-104S Starfighter starting from the position ($10000\,m$, $10000\,m$, $2500\,m$) related to the radar reference frame, in a level flight with velocity $v = 200\,m/s$ and heading $\chi = 0$, each of the maneuvers loop and the barrel roll is started a few seconds after the initial condition. The process noise variances were set as $\sigma^2_{\dot{\alpha}} = 0.15\ (rad/s)^2$, $\sigma^2_{\dot{\mu}} = 0.75\ (rad/s)^2$ and $\sigma^2_{\dot{\eta}} = 0.0001$. The figure 2 shows the results of tracking both maneuvers by the EKBF.

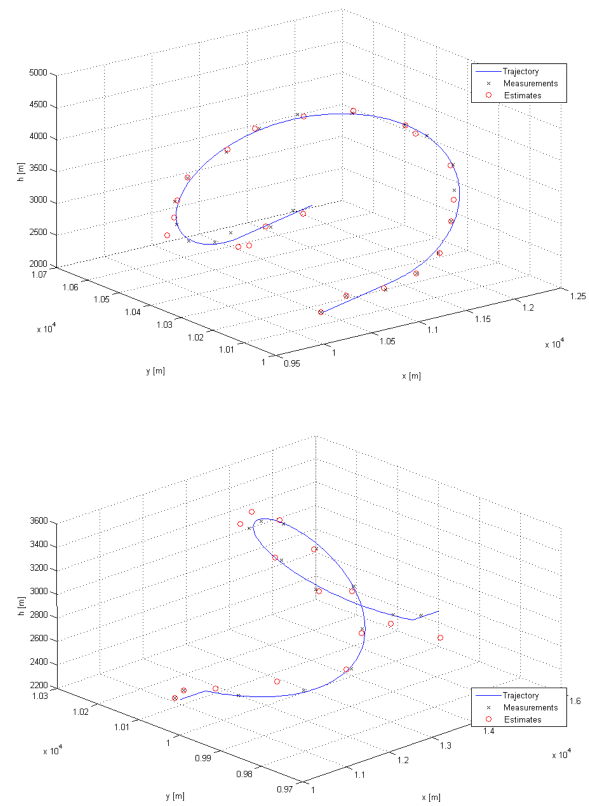

Figure 2: Tracking of loop and barrel roll maneuvers by the EKBF

The detailed modeling grants versatility to the filter as it provides dynamics to follow agile maneuvers. It is observable that for both maneuvers the tracking follows the trajectory closely ilustrating the effectiveness of the EKBF for air combat maneuvers.

For verifying proposed filters performance, results of application of the EKBF, EKBF-D and NEKBF to a validation trajectory are presented. The trajectory start from a condition of level flight in direction of x at cruise altitude (h_0), cruise velocity (v_0) and cruise angle of attack (α_0). At $t = 10\,s$ the aircraft model receives control inputs: a ramp of angle of attack at the maximum allowable rate $\dot{\alpha}_{max}$ and saturated at $\alpha_0 + \Delta\alpha$, and a ramp of bank angle at the maximum allowable rate $P_{max} = \dot{\mu}_{max}$ and saturated at $\Delta\mu$. The considered situation was an ascendant curve produced by control inputs of $\Delta\alpha = +2°$ and $\Delta\mu = +25°$. The figures 3 shows results of tracking for the ascendant maneuver of Cessna T-37A. For those results the filters were adjusted with $\sigma_{\dot{\alpha}}^2 = 0.0025\ (rad/s)^2$, $\sigma_{\dot{\mu}}^2 = 0.25\ (rad/s)^2$ and $\sigma_{\dot{\eta}}^2 = 0.01$.

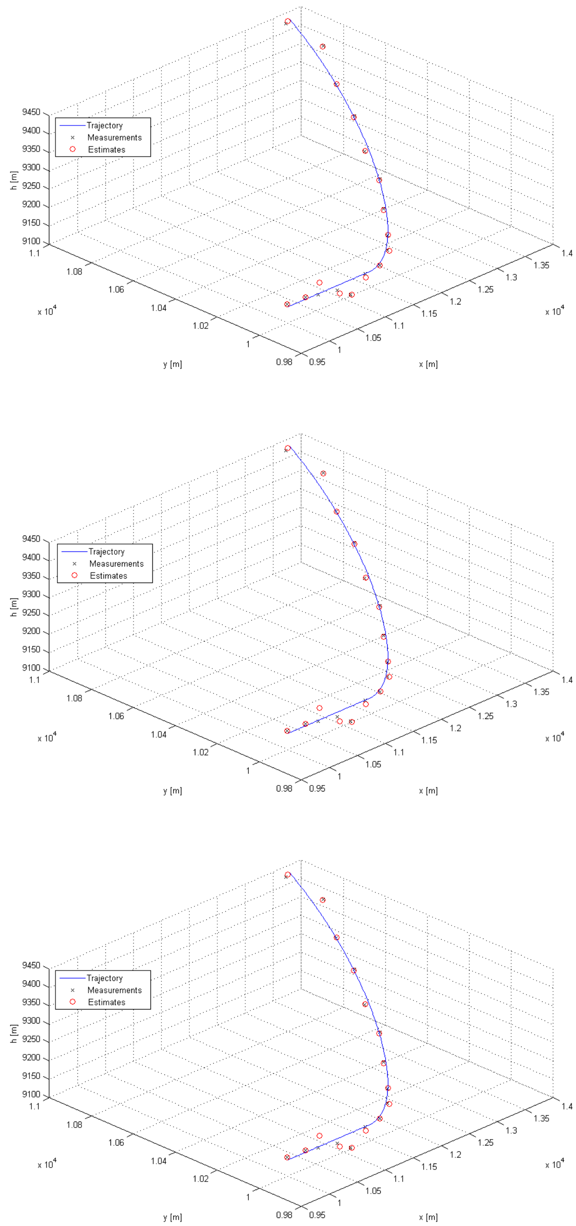

Figure 3: Tracking of the Cessna T-37A in ascendant curve by the EKBF, EKBF-D and NEKBF respectively

The EKBF, EKBF-D and NEKBF show high adhesion to the trajectories without any apparent divergence. In general, the estimate for $k = 3$ exhibits a significant error in altitude due to high errors on the initialized estimate ($\hat{\mathbf{x}}_3$ and $\mathbf{P}_3$).

For the ascendant maneuver performed by the Cessna T-37A, the performance indexes were evaluated. The figures 4 to 5 show the performance indexes. The in-

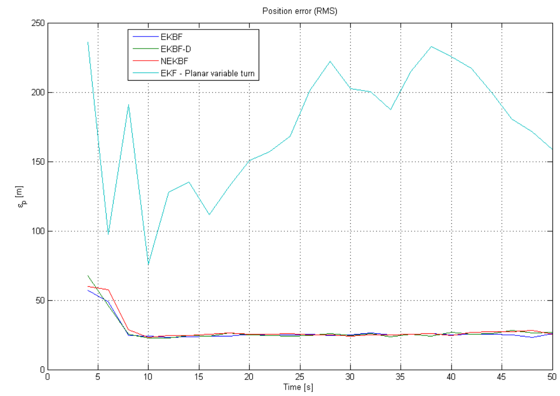

Figure 4: RMS errors of position for the Cessna T-37A maneuver

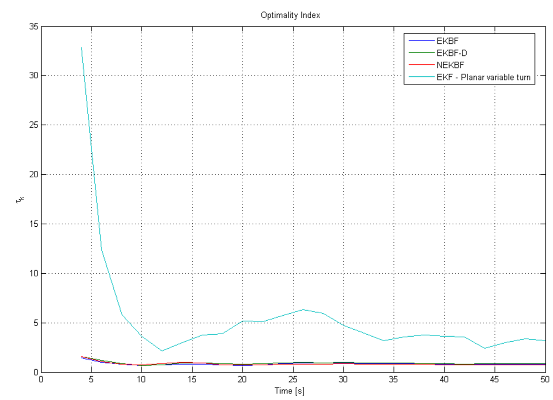

Figure 5: Optimality indexes

dexes were evaluated on a basis of $N = 100$ Monte Carlo runs.

The position errors of the filters EKBF, EKBF-D and NEKBF are of similar magnitude, and much smaller than the position error for the EKF based on a planar variable curve. The position errors are adequate for the proposed filters of this paper with a stabilized value of about $25\,m$.

A similar ascendant maneuver of the Cessna T-37A ($\Delta\alpha = +2°$ and $\Delta\mu = +20°$) was considered for evaluation of the robustness index.

In figure 6 the robustness index computed for the EKBF, EKBF-D and NEKBF is shown. In addition two reference limits are set for objective evaluation of filters robustness. The lower boundary of robustness is the threshold $\frac{1}{n}\sqrt{\text{trace}\{\mathbf{P}_k\}}$ evaluated for the EKBF as designed. The higher boundary is three times the previous threshold $3 \times \frac{1}{n}\sqrt{\text{trace}\{\mathbf{P}_k\}}$. Those limits allow the decision about the robustness of any filter: if the robustness index is lower than the lower boundary then definitely the filter is robust. However, if the index is higher than the higher boundary, the filter is not

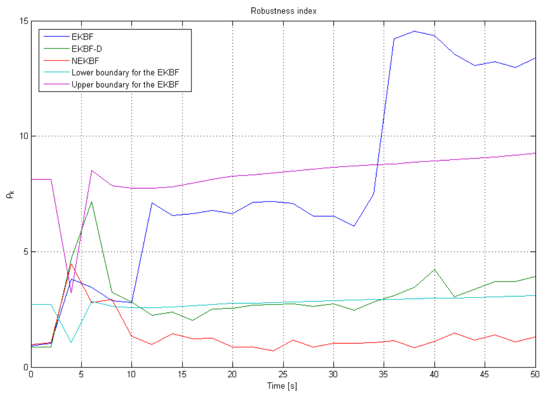

Figure 6: Robustness indexes

robust. Intermediary values of the index demands additional criteria for designating an estimator as robust.

The greatest degree of robustness is shown by the NEKBF, which can be definitely considered as robust. The filter EKBF-D shows a significant degree of robustness, with an index slightly lower than the lower boundary. According to the established criterion, it is not possible to assert that the EKBF is robust. This result demonstrates robustness against variation of parameters, and it is not necessarily related to the robustness against sensor degradation.

6 Conclusions

The limitations of well established models of the literature for target tracking were overcome by the use of the Miele model [5].

As an overall result, the designed filters presented qualities of high accuracy, small position errors, minimum influence of the nonlinearities on the performance, and versatility to a variety of maneuvers for different aircraft.

The methods of augmentation to the EKBF were found to be beneficial as they reduce the effects of nonlinearities on accuracy and increase robustness. Among the two augmented filters, the simplest is the EKBF-D which provides a stable index of optimality without increasing the computational complexity over the EKBF. The most elaborated filter is the NEKBF which increases the optimality and robustness of the EKBF at a cost of a greater implemention complexity.

Two major limitations were found for the designed filters: a maximum range of 30 km for which the tracking is effective due to the sensor characteristics (degeneration of measurement with the distance), and the minimum computational requirements for the execution of numerical integration. These limitations can be mitigated by two facts: (i) with techniques of registering measurements and bias compensation, the measurements degeneration in relation to the distance can be reduced; (ii) with adoption of defense systems with high computational capacity the proposed algorithms have wide applicability.

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